

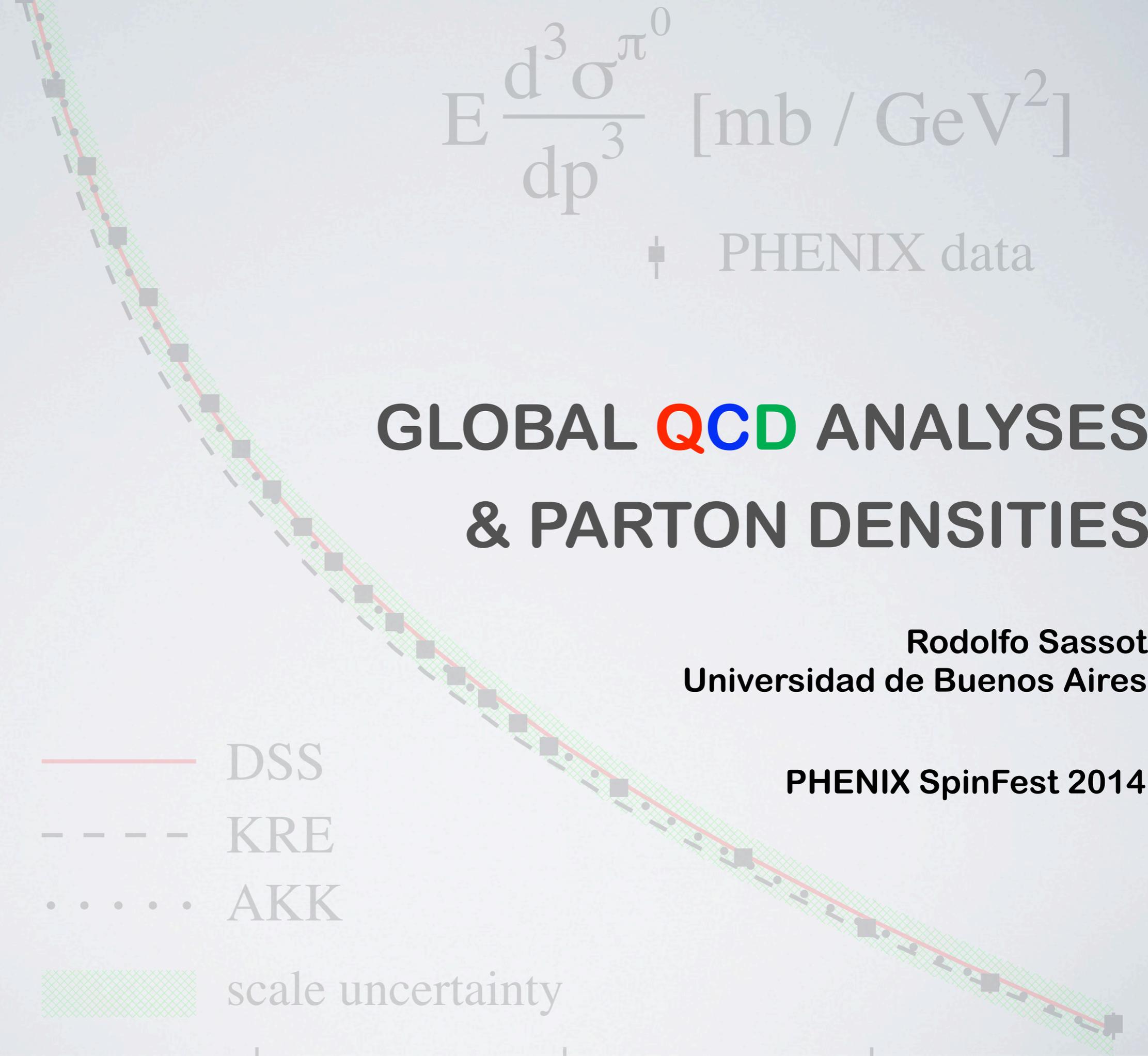
$$E \frac{d^3 \sigma_{\pi^0}}{dp^3} \text{ [mb / GeV}^2\text{]}$$

■ PHENIX data

# GLOBAL QCD ANALYSES & PARTON DENSITIES

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Universidad de Buenos Aires

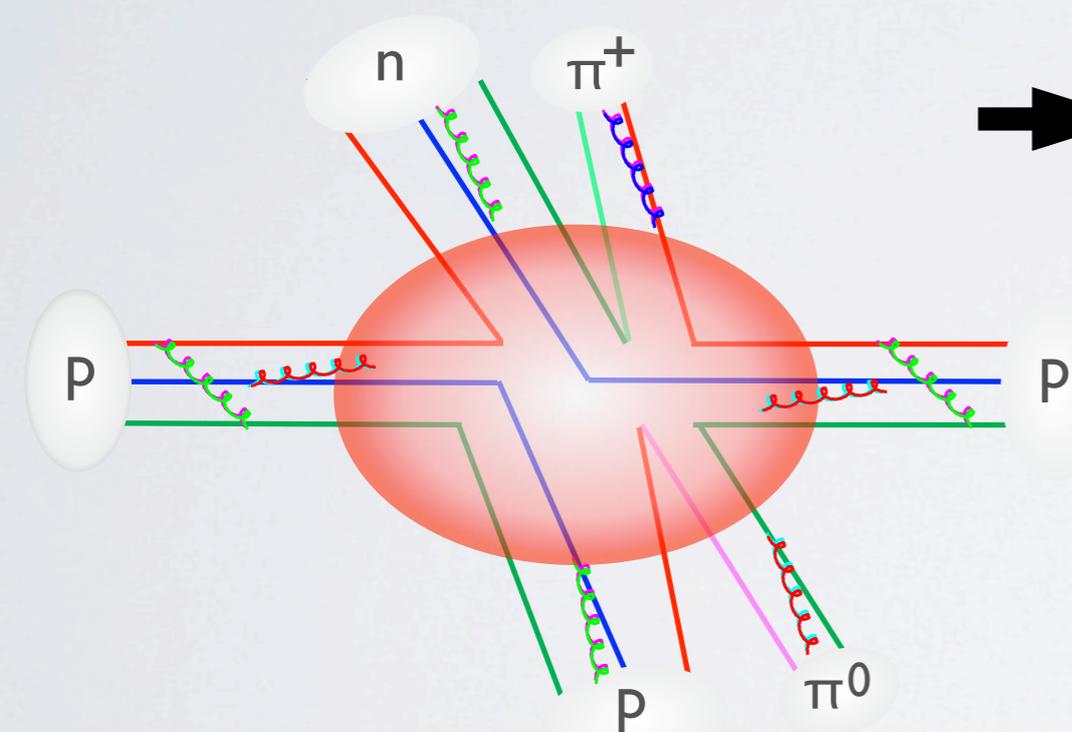
PHENIX SpinFest 2014



# why global analyses?

*why are they relevant for an experiment/experimentalist?*

→ only way to relate **measured cross sections**  
unambiguously to the **underlying physics**  
*(if we accelerate/detect hadrons)*



→ translate **hadronic degrees of freedom**  
into those of **quarks and gluons**  
*(and vice versa)*

→ combine/compare **hadronic data**  
from a **QCD** perspective

not the case at LEP ; tell the difference between



and



# what do we get/learn from a global analyses?

- ➔ PDFs “*parton distribution functions*”
  - what’s a PDFs?
  - how do we extract PDFs?

*probability densities?  
not quite, but close enough*

**First Lecture**
- ➔ FFs “*fragmentation functions*”
  - state of the art
  - perspective

*hadronization probabilities?*

**Second Lecture**
- ➔ pPDFs “*polarized PDFs*”
  - DSSV’08 DSSV’14 NNPDF’14
  - impact of forthcoming data

*helicity / longitudinal spin densities*

**Third Lecture**
- ➔ nPDFs “*nuclear PDFs*”
  - why?
  - nuclear issues

*PDFs in nuclei*

**Fourth Lecture**

# First Lecture:

*list of words we keep repeating*

1.1 pQCD:

1.2 running coupling constant:

1.3 factorization:

1.4 factorization scale:

1.5 scale dependence:

1.6 LO, NLO, NNLO, ...

1.7 global fits:

1.8 unpolarized PDFs:

1.9 PDFs & hadron structure:

*keep asking!!*

# I.I pQCD

QCD is “the” theory of strong interactions

very deep resemblance with QED: gauge symmetry

## NICE & EASY

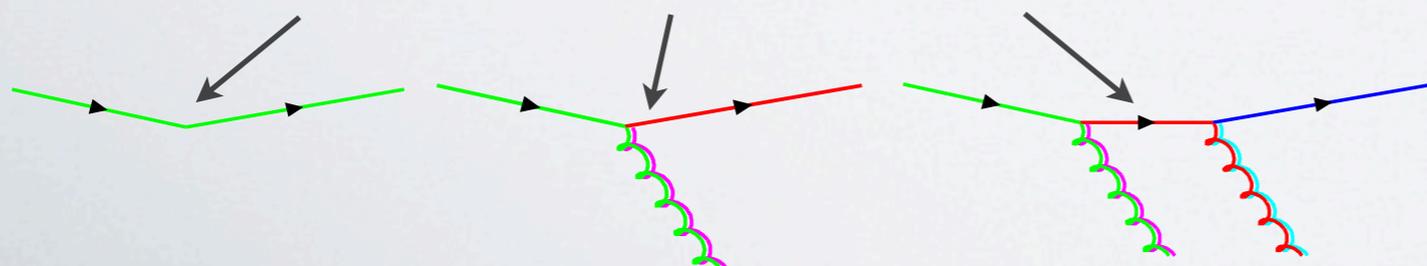
massless gluons ~ photons  
color charge ~ e charge  
weak coupling regime

## NOT EASY

gluons do carry color charges  
eight color (anti)charges  
weak at high energy

probing weak coupling regime

$$\sigma = \sigma^{(0)} + \alpha_s \sigma^{(1)} + \alpha_s^2 \sigma^{(2)} + \alpha_s^3 \sigma^{(3)} + \dots$$

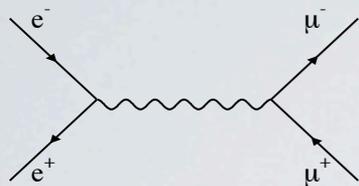


*perturbative expansion*

# 1.2 running coupling constant

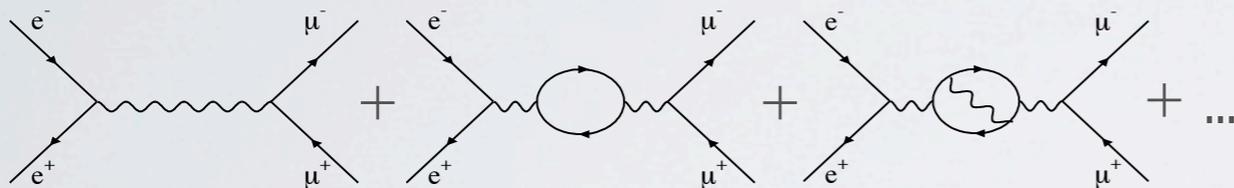
even in QED the coupling “runs”

but we don't notice...

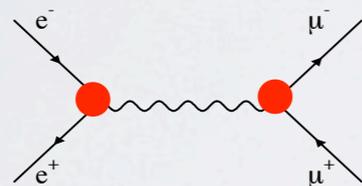


$$\sigma = \frac{4\pi}{3s} \alpha_0^2$$

$$s \equiv (p_{e^-} + p_{e^+})^2$$



$$\sigma = \frac{4\pi}{3s} \alpha_0^2 (1 + \alpha_0 \sigma^{(1)}(s) + \alpha_0^2 \sigma^{(2)}(s) + \dots)$$

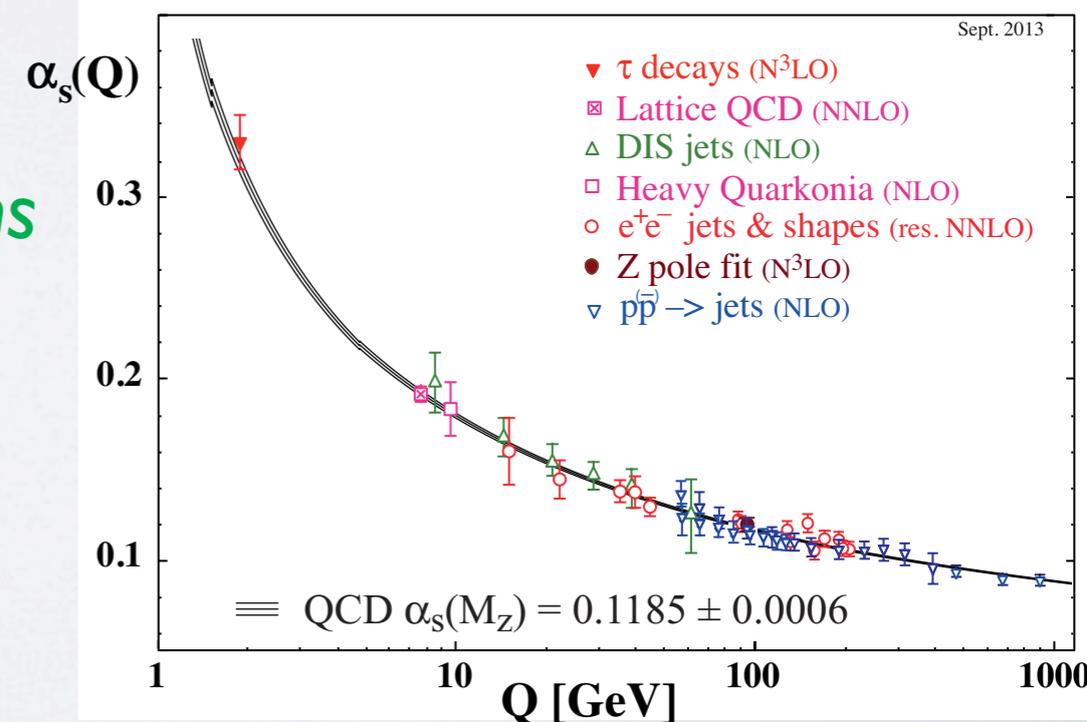


$$\sigma = \frac{4\pi}{3s} \alpha_{\text{eff}}^2(\underline{s})$$

effective running coupling “accounts” for corrections

resolving power: energy - wave length - distance

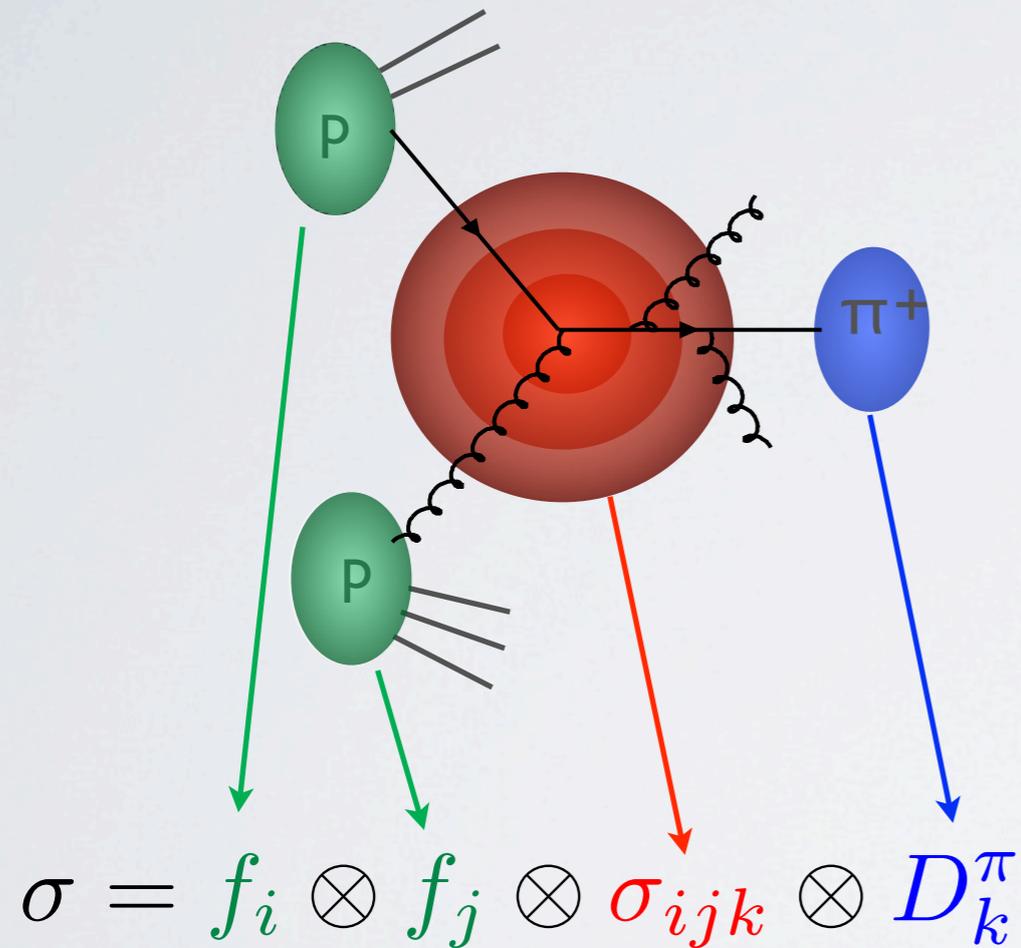
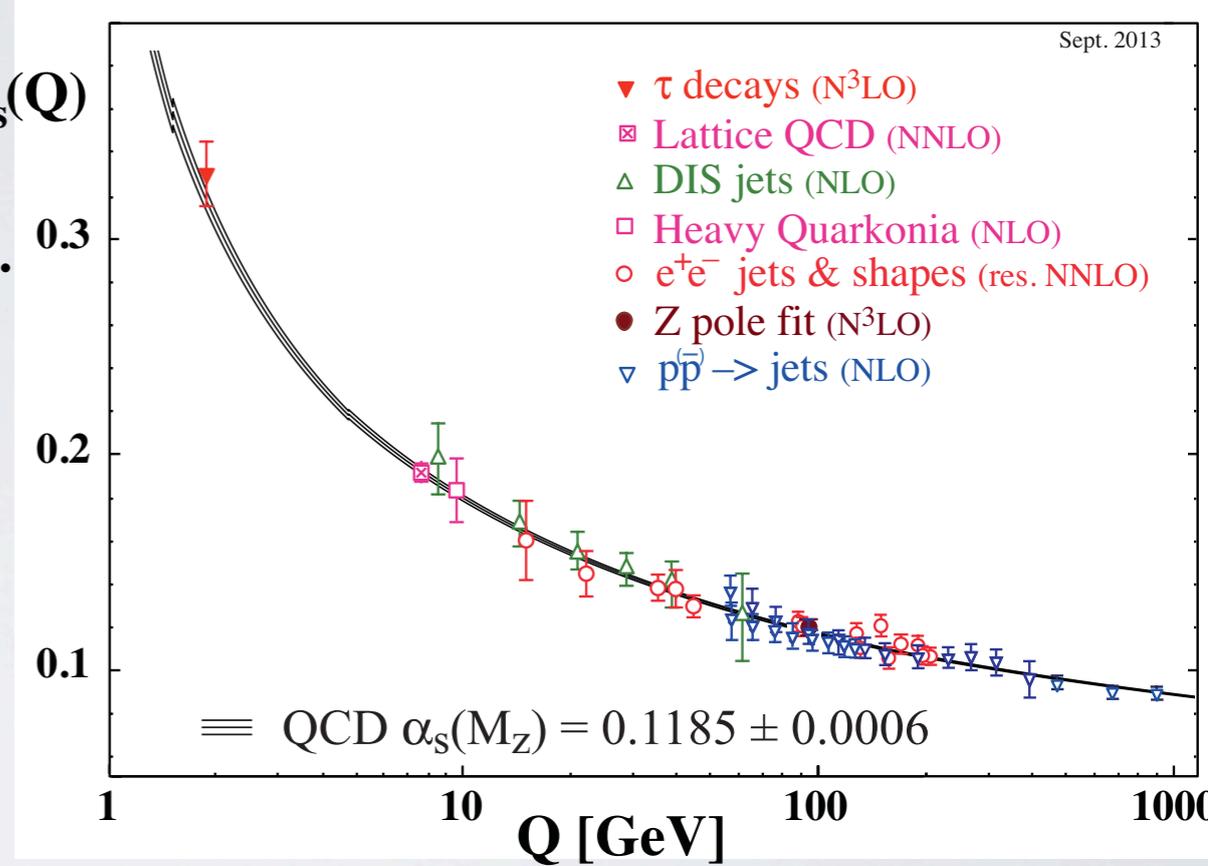
$$\lambda \sim \frac{\hbar c}{Q} \simeq \frac{0.2 [\text{GeV fm}]}{Q}$$



$\sim 0.2 \text{ fm} \quad \sim 0.02 \text{ fm}$

# 1.3 factorization

$$\sigma = \sigma^{(0)} + \alpha_s \sigma^{(1)} + \alpha_s^2 \sigma^{(2)} + \alpha_s^3 \sigma^{(3)} + \dots$$



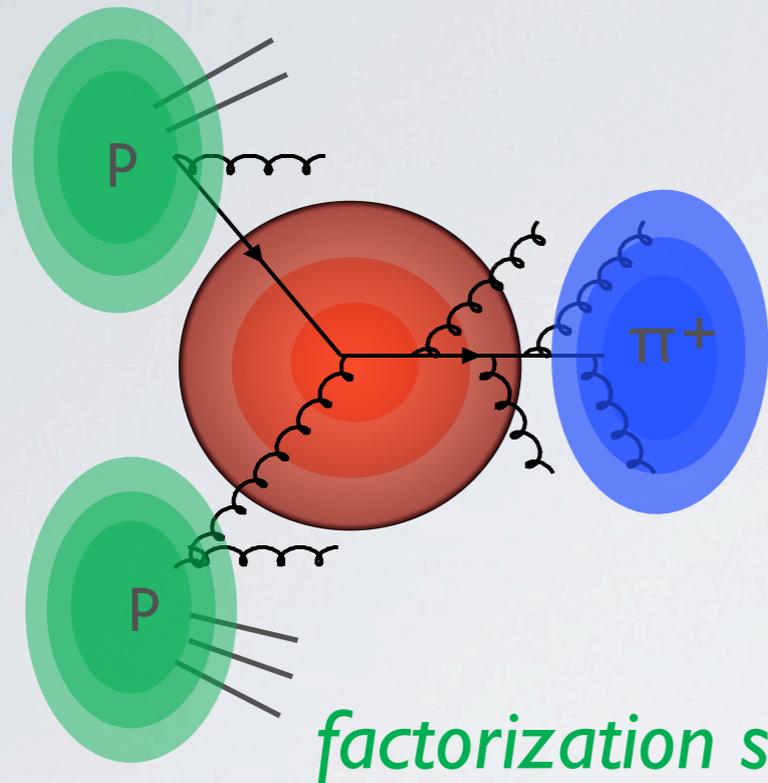
$$D_k^\pi(z) \quad z \simeq \frac{P_\pi}{p_k}$$

**FFs** (fragmentation functions)

$$\sigma_{ijk} = \sigma_{ijk}^{(0)} + \alpha_s \sigma_{ijk}^{(1)} + \dots \quad (\text{hard / partonic xsection})$$

(parton distribution functions)  $f_i(x) \quad x \simeq \frac{p_i}{P} \quad \text{collinear}$

# 1.4 factorization scale



$$\sigma = f_i \otimes f_j \otimes \sigma_{ijk} \otimes D_k^\pi$$

$\sigma_{ijk}(p_T^2)$

$f_i(x, p_T^2)$        $D_k^\pi(z, p_T^2)$

*factorization scale: how much goes in each piece*

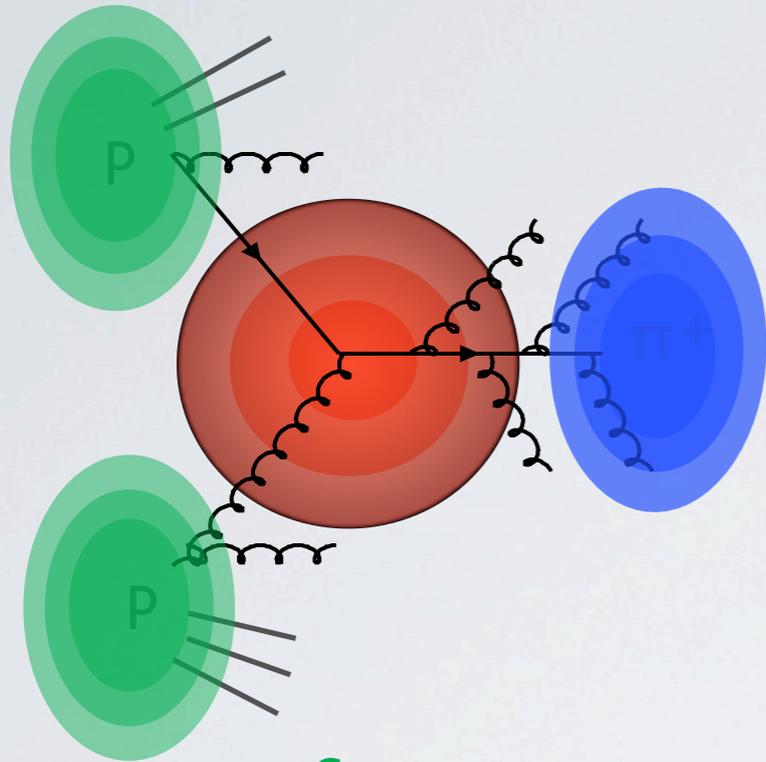
- ➔ usual choice: most relevant scale of the processes
- ➔ in an “all order calculation,” the choice is irrelevant
- ➔ if not: theory error ~~estimate~~ indication

# 1.5 scale dependence

Altarelli-Parisi (DGLAP) equations

$$\frac{\partial D_k^\pi(z, p_T^2)}{\partial \log(p_T^2)} = \hat{\sigma}_{ijk} \otimes D_j^\pi(z, p_T^2)$$

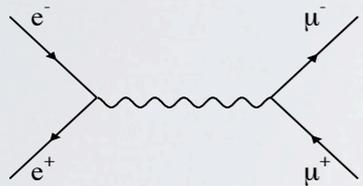
$$\sigma_{ijk}(p_T^2) \longrightarrow \sigma_{ijk}^0$$



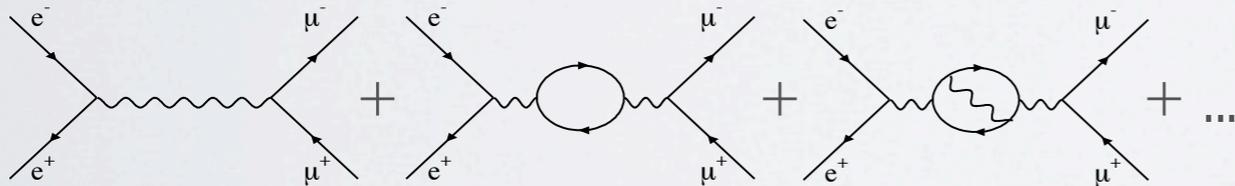
$$\sigma = f_i \otimes f_j \otimes \sigma_{ijk} \otimes D_k^\pi$$

$\downarrow$   $f_i(x, \underline{p_T^2})$        $\downarrow$   $D_k^\pi(z, \underline{p_T^2})$

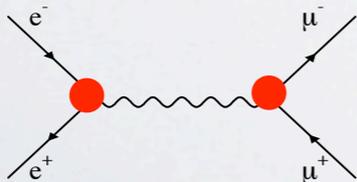
factorization scale: how much goes in each piece



$$\sigma = \frac{4\pi}{3s} \alpha_0^2$$



$$\sigma = \frac{4\pi}{3s} \alpha_0^2 (1 + \alpha_0 \sigma^{(1)}(s) + \alpha_0^2 \sigma^{(2)}(s) + \dots)$$



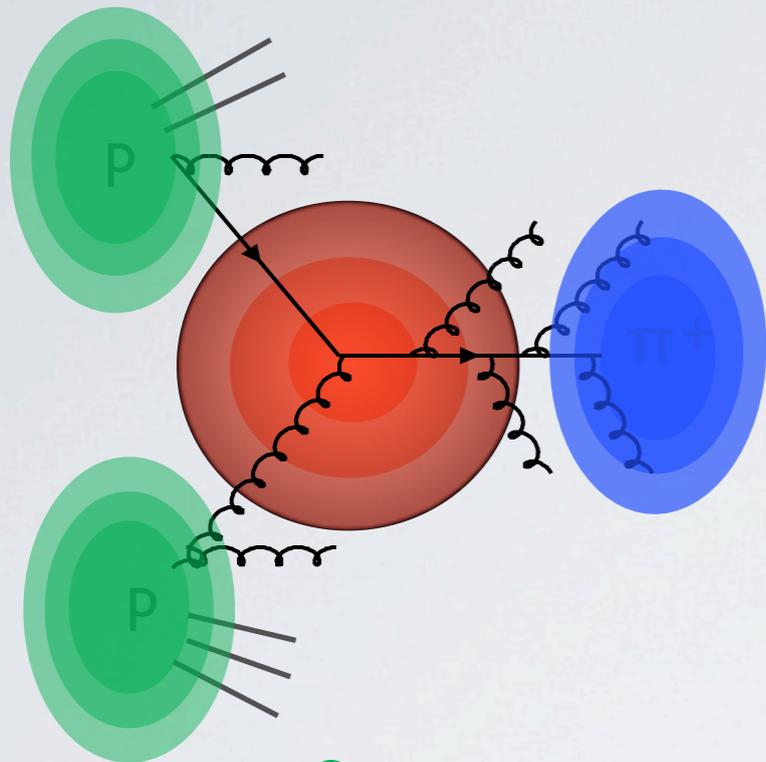
$$\sigma = \frac{4\pi}{3s} \alpha_{\text{eff}}^2(\underline{s})$$

# 1.6 LO, NLO, NNLO...

Altarelli-Parisi (DGLAP) equations

$$\frac{\partial D_k^\pi(z, p_T^2)}{\partial \log(p_T^2)} = \hat{\sigma}_{ijk} \otimes D_j^\pi(z, p_T^2)$$

$$\sigma_{ijk}(p_T^2) \longrightarrow \sigma_{ijk}^0$$



$$\sigma = f_i \otimes f_j \otimes \sigma_{ijk} \otimes D_k^\pi$$

$$f_i(x, \underline{p_T^2})$$

$$D_k^\pi(z, \underline{p_T^2})$$

factorization scale: how much goes in each piece

## Bonus:

running pdfs +  $\sigma_{ijk}^{(0)}$

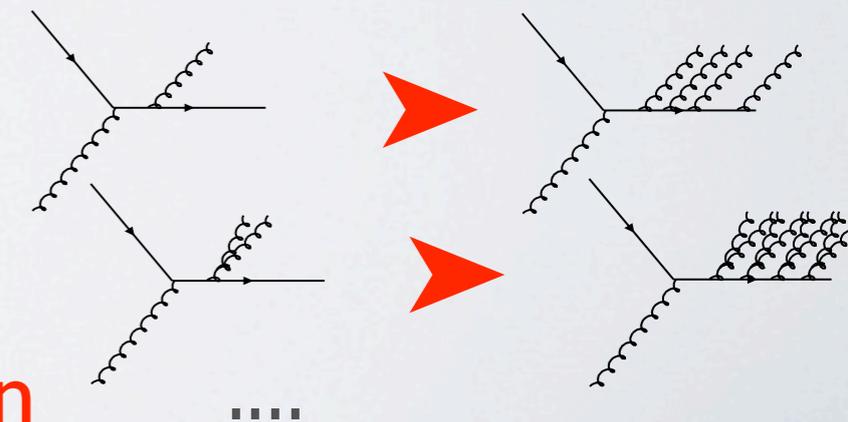
➤ LO resummation

running pdfs +  $\sigma_{ijk}^{(1)}$

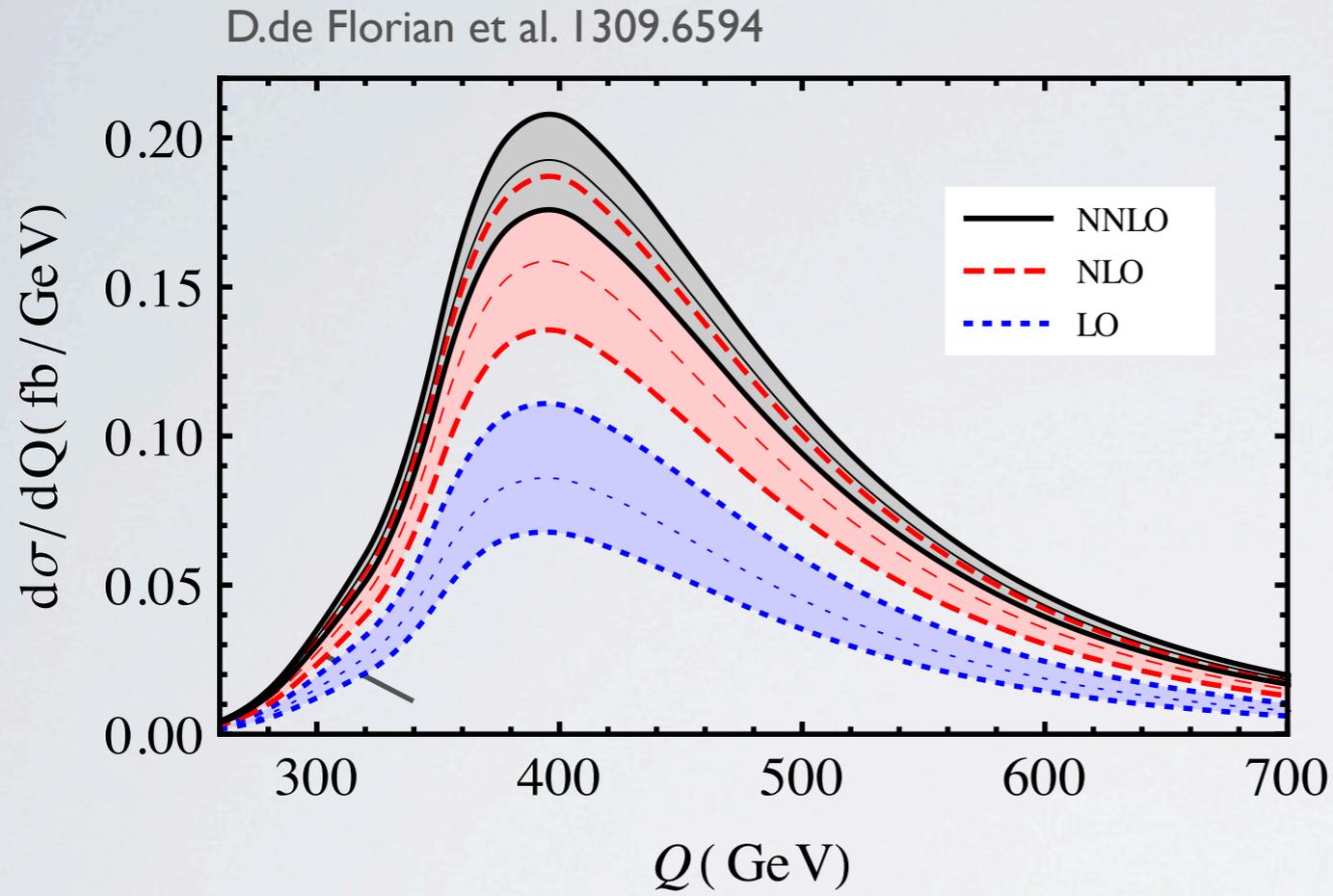
➤ NLO resummation

running pdfs +  $\sigma_{ijk}^{(2)}$

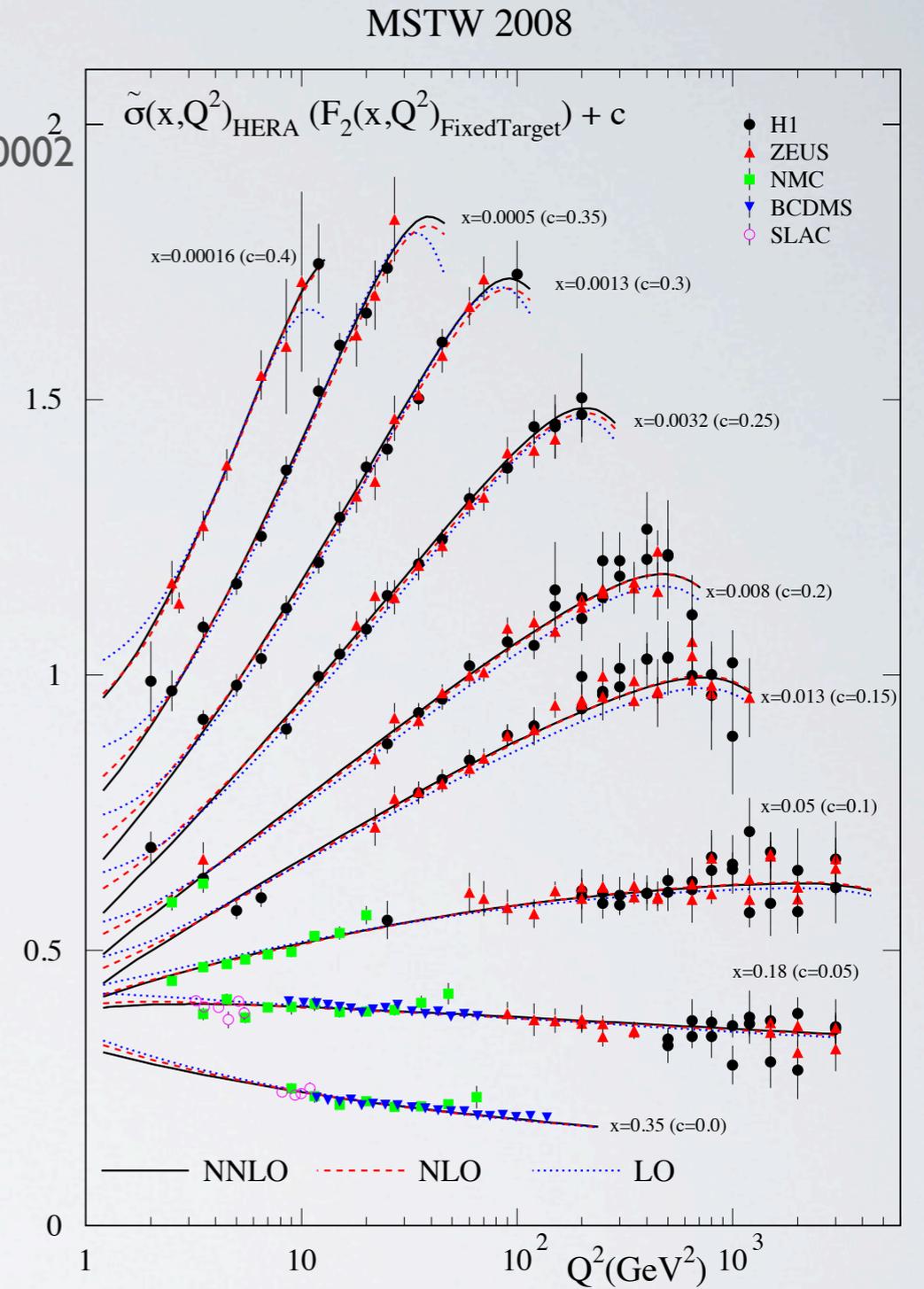
➤ NNLO resummation



# 1.6 LO, NLO, NNLO...



A. Martin et al 0901.0002



# 1.7 global fits

perturbation theory  $\longrightarrow$   $\frac{\partial D_k^\pi(z, p_T^2)}{\partial \log(p_T^2)} = \hat{\sigma}_{ijk} \otimes D_j^\pi(z, p_T^2)$

data  $\longrightarrow$   $D_j^\pi(z, p_{T0}^2) ??$

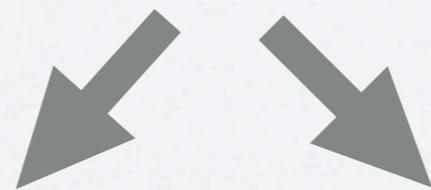
*not so easy...*

★ different  $z \sim$  different  $p_T^2$

★  $\sigma(z, p_T^2) = \sigma_i \otimes D_i^\pi = \sum_i \int_z^1 \frac{dy}{y} \sigma_i(y) D_i^\pi\left(\frac{z}{y}, p_T^2\right)$

↑ all flavors      ↙ range in  $z$

★ limited range in  $z$ : inter/extra/polations



“traditional”  $\sim$  pre-defined parameterization      “NNPDF”  $\sim$  Monte-Carlo / Neural Networks

# 1.7 global fits: traditional approach

1. compute QCD  $\hat{\sigma}_{i..jk}$  corrections for measured processes

$$\begin{aligned}\sigma &= \sigma_i \otimes D_i^\pi \\ \sigma &= f_i \otimes \hat{\sigma}_{ij} \otimes D_j^\pi \\ \sigma &= f_i \otimes f_j \otimes \hat{\sigma}_{ijk} \otimes D_k^\pi\end{aligned}$$

at a given (same!) order  
same factorization conventions  
fast code

2. “model” for the PDFs/FFs:

$$D_i^\pi(z, p_{T0}^2) = N_i z^{\alpha_i} (1-z)^{\beta_i} (1 + \gamma_i z^{\delta_i} + \dots)$$

3. evolution to exp. scale

$$D_i^\pi(z, p_{T0}^2) \longrightarrow D_i^\pi(z, p_{Texp}^2)$$

4. compute xsection  $\sim 10^3$

$\sum_{data}$

$$\sigma_{exp} \stackrel{?}{=} \sigma[D_i^\pi(z, p_{Texp}^2)]$$

5.  $\sim 10^4$  compare & fit

$\chi^2 \sim OK$

$\chi^2 \neq OK$

*pro*: pre-defined param. allows fast computations  $\sim$  Mellin tricks

*contra*: ignore bias from pre-defined param.  $\sim$  inflate errors a posteriori

# 1.7 global fits: neural network approach

1. generate monte-carlo replicas of the data ( $\sim 1000$ )  
replicas fluctuate around the original data
2. generate the PDFs with neural net  
total flexibility
3. fit the data replicas  
get as many PDFs as replicas
4. expectations as monte carlo averages over PDFs ensemble
5. uncertainties as standard deviations

*pro:* unbiased distributions, statistically sound  $\sim$  uncertainties

*contra:* very hard to go beyond DIS  $\sim$  reweighting

# 1.8 unpolarized PDFs:

D.Duke, J.Owens, “ $Q^2$  Dependent Parametrizations of parton distribution functions”

Phys.Rev.D30, 49 | 1984

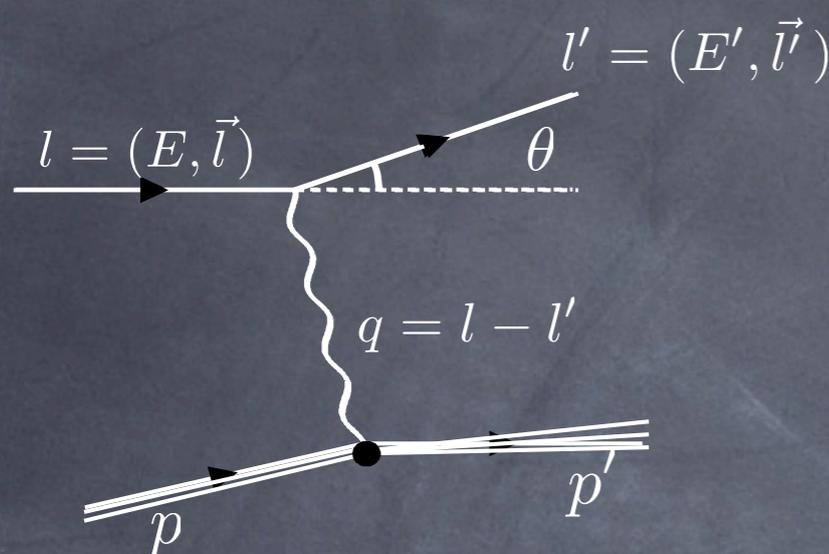
⋮

J. Gao, P. Nadolsky “A meta-analysis of parton distribution functions”

arXiv:1401.0013 | 2014

30  
years!

## DIS & Naive parton model



$$Q^2 \equiv -q^2 \simeq 4EE' \sin^2(\theta/2) \quad \nu \equiv E - E' = \frac{q \cdot p}{M}$$

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha_{em}^2}{Q^4} \frac{E'}{E} \left\{ \cos^2 \frac{\theta}{2} + \frac{Q^2}{2M^2} \sin^2 \frac{\theta}{2} \right\} \quad \text{elastic}$$

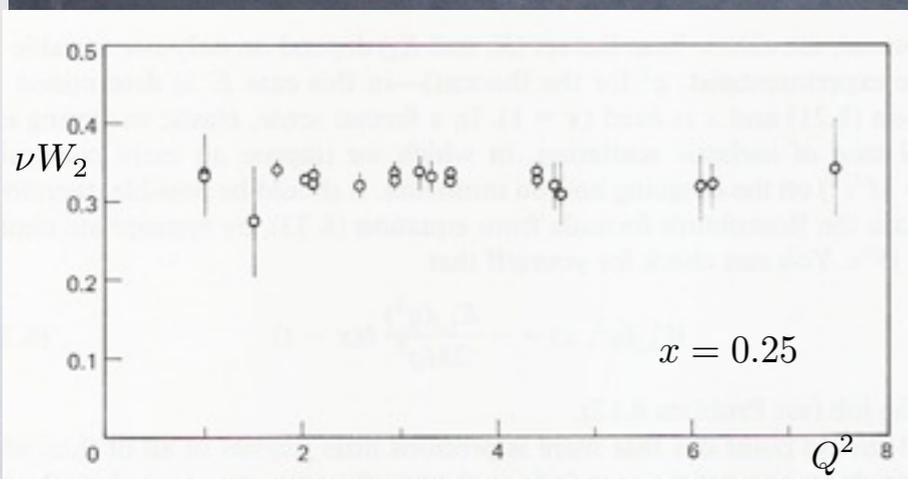
$$\frac{d^2\sigma}{dQ^2 d\nu} = \frac{4\pi\alpha_{em}^2}{Q^4} \frac{E'}{E} \left\{ W_2(Q^2, \nu) \cos^2 \frac{\theta}{2} + 2W_1(Q^2, \nu) \sin^2 \frac{\theta}{2} \right\}$$

$$W_2^{el}(Q^2, \nu) = e_q^2 \delta\left(\nu - \frac{Q^2}{2M}\right) \quad W_1^{el}(Q^2, \nu) = e_q^2 \frac{Q^2}{4M^2} \delta\left(\nu - \frac{Q^2}{2M}\right)$$

$$W_2(Q^2, \nu) = \sum_i \int_0^1 dy_i f_i(y_i) e_i^2 \delta\left(\nu - \frac{Q^2}{2y_i M}\right)$$

$$= \sum_i e_i^2 f_i(x) \frac{x}{\nu}$$

$$F_2(x) \equiv \nu W_2 = \sum_i e_i^2 x f_i(x)$$



# I.8 unpolarized PDFs:

D.Duke, J.Owens, “ $Q^2$  Dependent Parametrizations of parton distribution functions”

Phys.Rev.D30, 49 | 1984

⋮

J. Gao, P. Nadolsky “A meta-analysis of parton distribution functions”

arXiv:1401.0013 | 2014

30  
years!

## DIS & Naive parton model

$$\frac{1}{x} F_2^{ep}(x) = \left(\frac{2}{3}\right)^2 [u(x) + \bar{u}(x)] + \left(\frac{1}{3}\right)^2 [d(x) + \bar{d}(x)] + \left(\frac{1}{3}\right)^2 [s(x) + \bar{s}(x)]$$

valence and sea quarks

$$u_{sea}(x) = \bar{u}_{sea}(x)$$

$$u_{val}(x) = u(x) - \bar{u}_{sea}(x)$$

charge, baryon number, strangeness

$$\int_0^1 dx u_{val}(x) = 2 \quad \int_0^1 dx d_{val}(x) = 1$$

$$\int_0^1 dx s_{val}(x) = 0$$

isospin

$$u_{val}^p(x) = d_{val}^n(x) \quad d_{val}^p(x) = u_{val}^n(x)$$

gluons

$$\int_0^1 dx x(u + \bar{u} + d + \bar{d} + s + \bar{s}) = 1$$

0.36 + 0.18 + 0.00 = 0.54

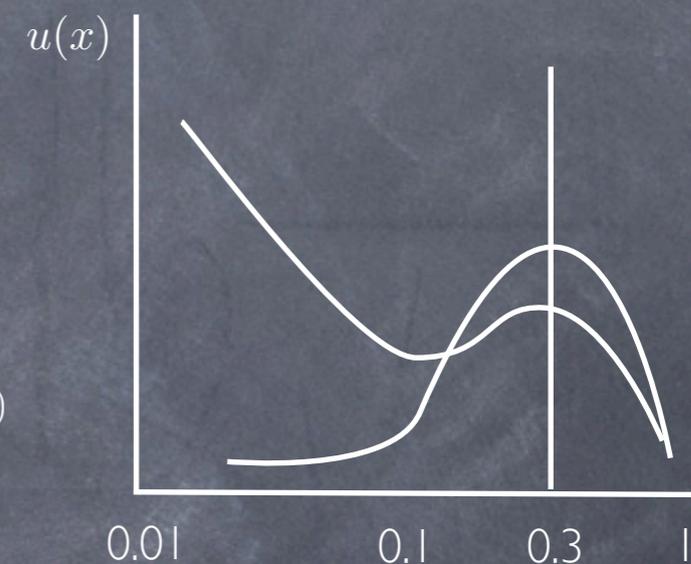
one parton

$$u(x) \sim \delta(1-x)$$

three partons

$$u(x) \sim \delta(1/3-x)$$

interactions ~ smearing



# 1.8 unpolarized PDFs:

D.Duke, J.Owens, “ $Q^2$  Dependent Parametrizations of parton distribution functions”

Phys.Rev.D30, 49 | 1984

⋮

J. Gao, P. Nadolsky “A meta-analysis of parton distribution functions”

arXiv:1401.0013 | 2014

30  
years!

Fixed target :  
DIS and DY

HERA

Tevatron

Process	Subprocess	Partons	$x$ range
$l^\pm \{p, n\} \rightarrow l^\pm X$	$\gamma^* q \rightarrow q$	$q, \bar{q}, g$	$x \gtrsim 0.01$
$l^\pm n/p \rightarrow l^\pm X$	$\gamma^* d/u \rightarrow d/u$	$d/u$	$x \gtrsim 0.01$
$pp \rightarrow \mu^+ \mu^- X$	$u\bar{u}, d\bar{d} \rightarrow \gamma^*$	$\bar{q}$	$0.015 \lesssim x \lesssim 0.35$
$pn/pp \rightarrow \mu^+ \mu^- X$	$(u\bar{d})/(u\bar{u}) \rightarrow \gamma^*$	$\bar{d}/\bar{u}$	$0.015 \lesssim x \lesssim 0.35$
$\nu(\bar{\nu}) N \rightarrow \mu^-(\mu^+) X$	$W^* q \rightarrow q'$	$q, \bar{q}$	$0.01 \lesssim x \lesssim 0.5$
$\nu N \rightarrow \mu^- \mu^+ X$	$W^* s \rightarrow c$	$s$	$0.01 \lesssim x \lesssim 0.2$
$\bar{\nu} N \rightarrow \mu^+ \mu^- X$	$W^* \bar{s} \rightarrow \bar{c}$	$\bar{s}$	$0.01 \lesssim x \lesssim 0.2$
$e^\pm p \rightarrow e^\pm X$	$\gamma^* q \rightarrow q$	$g, q, \bar{q}$	$0.0001 \lesssim x \lesssim 0.1$
$e^+ p \rightarrow \bar{\nu} X$	$W^+ \{d, s\} \rightarrow \{u, c\}$	$d, s$	$x \gtrsim 0.01$
$e^\pm p \rightarrow e^\pm c\bar{c} X$	$\gamma^* c \rightarrow c, \gamma^* g \rightarrow c\bar{c}$	$c, g$	$0.0001 \lesssim x \lesssim 0.01$
$e^\pm p \rightarrow \text{jet} + X$	$\gamma^* g \rightarrow q\bar{q}$	$g$	$0.01 \lesssim x \lesssim 0.1$
$p\bar{p} \rightarrow \text{jet} + X$	$gg, qg, qq \rightarrow 2j$	$g, q$	$0.01 \lesssim x \lesssim 0.5$
$p\bar{p} \rightarrow (W^\pm \rightarrow l^\pm \nu) X$	$ud \rightarrow W, \bar{u}\bar{d} \rightarrow W$	$u, d, \bar{u}, \bar{d}$	$x \gtrsim 0.05$
$p\bar{p} \rightarrow (Z \rightarrow l^+ l^-) X$	$uu, dd \rightarrow Z$	$d$	$x \gtrsim 0.05$

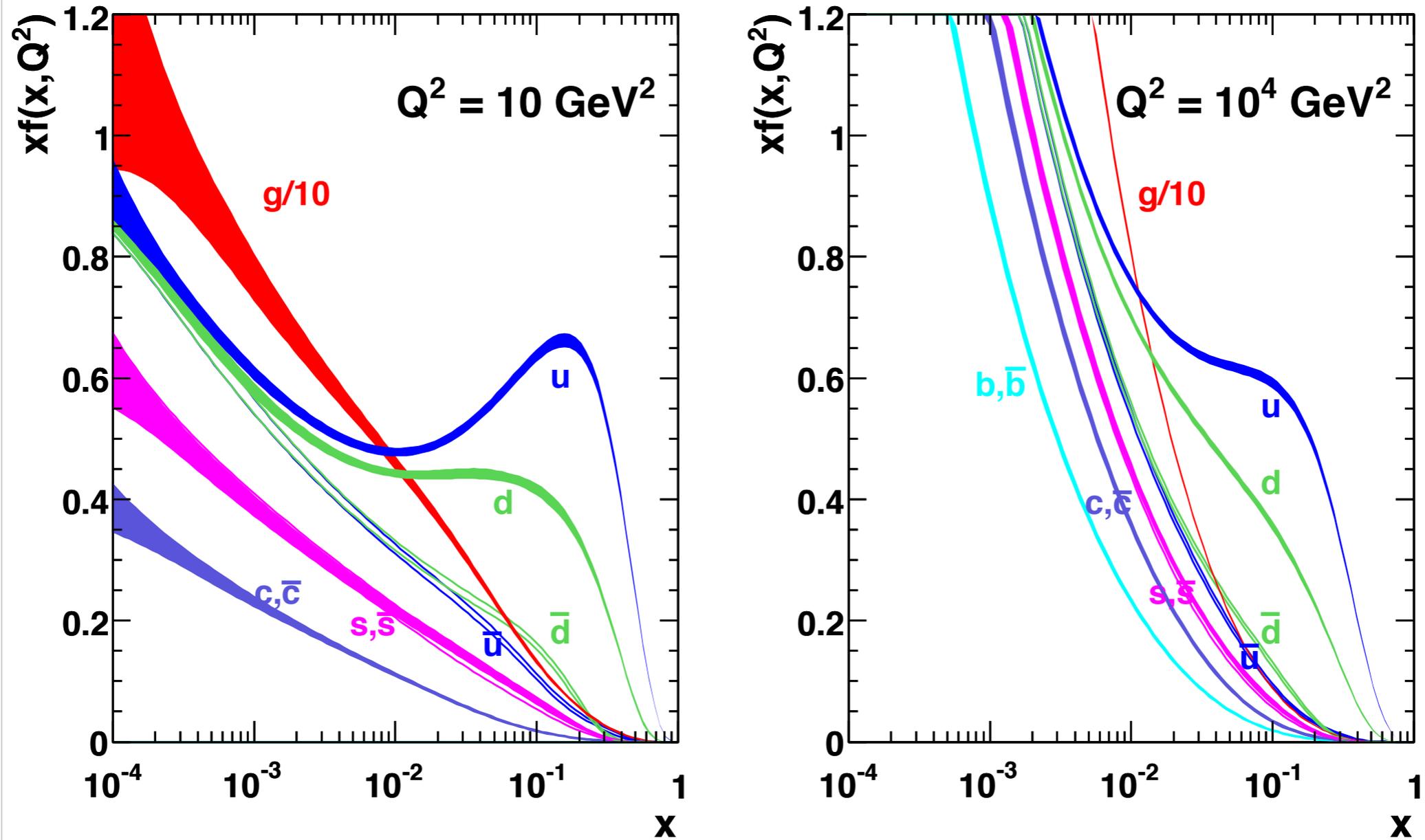
# 1.8 unpolarized PDFs:

set	order	data	$\alpha_s(M_Z)$	uncertainty	HQ	
MSTW 2008	NNLO	global	fitted (+ external variations)	Hessian (dynamical tolerance)	GM-VFN (ACOT +TR')	Martin, Stirling, Thorne, Watt
CT10	NNLO	global combined HERA	external (several values & older fit)	Hessian (dynamical tolerance)	GM-VFN (SACOT-X)	CTEQ, Lai et al.
NNPDF 2.1	NNLO	global combined HERA	external (several values & recent fit)	Monte Carlo (pdf replicas)	GM-VFN (FONLL)	NNPDF, Ball et al.
AB(K)M	NNLO	DIS+DY(f.t.)	fitted	Hessian	FFN +matching	Alekhin, Blümlein, Klein, Moch
(G)JR	NNLO	DIS+DY(f.t.)+ some jet	fitted	Hessian	FFN (VFN massless)	Glück, Jimenez Delgado, Reya
HERA PDF	NNLO	only DIS HERA	external	Hessian	GM-VFN (ACOT +TR')	H1 & Zeus collaborations

Each group provides a number of sets to compute central values and pdfs, pdf+coupling uncertainties

# 1.8 unpolarized PDFs:

MSTW 2008 NLO PDFs (68% C.L.)



# 1.8 unpolarized PDFs:

	$\chi^2/\text{dof}$
LO	1.180
NLO	0.942
NNLO	0.948

Data set	LO	NLO	NNLO
BCDMS $\mu p F_2$ [?]	165 / 153	182 / 163	170 / 163
BCDMS $\mu d F_2$ [?]	162 / 142	190 / 151	188 / 151
NMC $\mu p F_2$ [?]	137 / 115	121 / 123	115 / 123
NMC $\mu d F_2$ [?]	120 / 115	102 / 123	93 / 123
NMC $\mu n/\mu p$ [?]	131 / 137	130 / 148	135 / 148
E665 $\mu p F_2$ [?]	59 / 53	57 / 53	63 / 53
E665 $\mu d F_2$ [?]	49 / 53	53 / 53	63 / 53
SLAC $ep F_2$	24 / 18	30 / 37	31 / 37
SLAC $ed F_2$	12 / 18	30 / 38	26 / 38
NMC/BCDMS/SLAC $F_L$	28 / 24	38 / 31	32 / 31
E866/NuSea $pp$ DY [?]	239 / 184	228 / 184	237 / 184
E866/NuSea $pd/pp$ DY [?]	14 / 15	14 / 15	14 / 15
NuTeV $\nu N F_2$ [?]	49 / 49	49 / 53	46 / 53
CHORUS $\nu N F_2$ [?]	21 / 37	26 / 42	29 / 42
NuTeV $\nu N xF_3$ [?]	62 / 45	40 / 45	34 / 45
CHORUS $\nu N xF_3$ [?]	44 / 33	31 / 33	26 / 33
CCFR $\nu N \rightarrow \mu\mu X$ [?]	63 / 86	66 / 86	69 / 86
NuTeV $\nu N \rightarrow \mu\mu X$ [?]	44 / 40	39 / 40	45 / 40
H1 MB 99 $e^+p$ NC [?]	9 / 8	9 / 8	7 / 8
H1 MB 97 $e^+p$ NC [?]	46 / 64	42 / 64	51 / 64
H1 low $Q^2$ 96–97 $e^+p$ NC [?]	54 / 80	44 / 80	45 / 80
H1 high $Q^2$ 98–99 $e^-p$ NC [?]	134 / 126	122 / 126	124 / 126
H1 high $Q^2$ 99–00 $e^+p$ NC [?]	153 / 147	131 / 147	133 / 147
ZEUS SVX 95 $e^+p$ NC [?]	35 / 30	35 / 30	35 / 30
ZEUS 96–97 $e^+p$ NC [?]	118 / 144	86 / 144	86 / 144
ZEUS 98–99 $e^-p$ NC [?]	61 / 92	54 / 92	54 / 92
ZEUS 99–00 $e^+p$ NC [?]	75 / 90	63 / 90	65 / 90
H1 99–00 $e^+p$ CC [?]	28 / 28	29 / 28	29 / 28
ZEUS 99–00 $e^+p$ CC [?]	36 / 30	38 / 30	37 / 30
H1/ZEUS $ep F_2^{\text{charm}}$	110 / 83	107 / 83	95 / 83
H1 99–00 $e^+p$ incl. jets [?]	109 / 24	19 / 24	—
ZEUS 96–97 $e^+p$ incl. jets [?]	88 / 30	30 / 30	—
ZEUS 98–00 $e^\pm p$ incl. jets [?]	102 / 30	17 / 30	—
DØ II $p\bar{p}$ incl. jets [?]	193 / 110	114 / 110	123 / 110
CDF II $p\bar{p}$ incl. jets [?]	143 / 76	56 / 76	54 / 76
CDF II $W \rightarrow \ell\nu$ asym. [?]	50 / 22	29 / 22	30 / 22
DØ II $W \rightarrow \ell\nu$ asym. [?]	23 / 10	25 / 10	25 / 10
DØ II $Z$ rap. [?]	25 / 28	19 / 28	17 / 28
CDF II $Z$ rap. [?]	52 / 29	49 / 29	50 / 29
All data sets	<b>3066 / 2598</b>	<b>2543 / 2699</b>	<b>2480 / 2615</b>

# 1.9 PDFs & hadron structure:

*a word of caution*

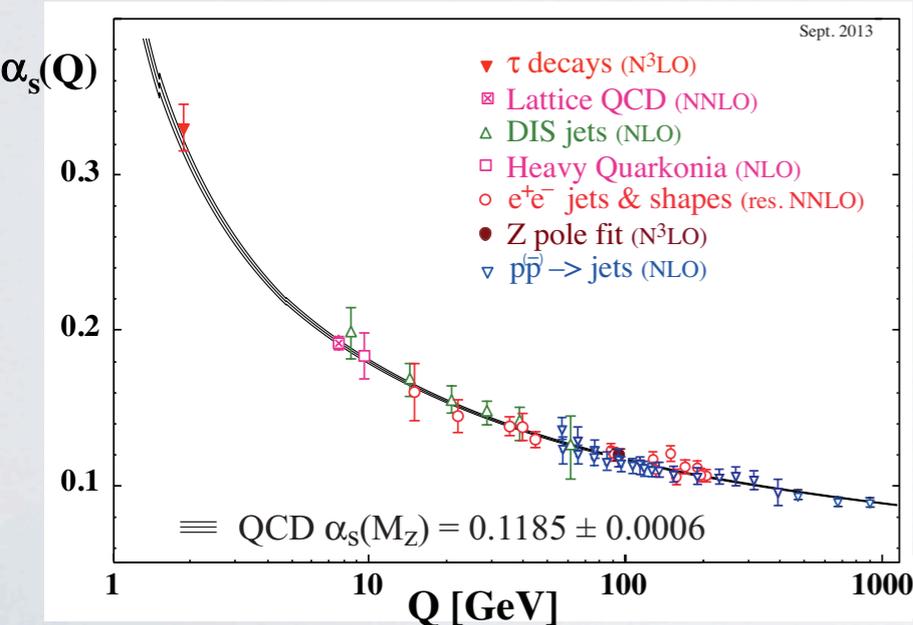
well defined pieces of measured cross sections,  
(rigorous F.T. definitions in terms of quark and gluon fields)

PDFs: universal, *process independent*

well determined, *uncertainties under control*

not just a  
theory  
artifact

*but certainly not naive parton probabilities (of a non interacting picture)*



*QCD interactions are there*

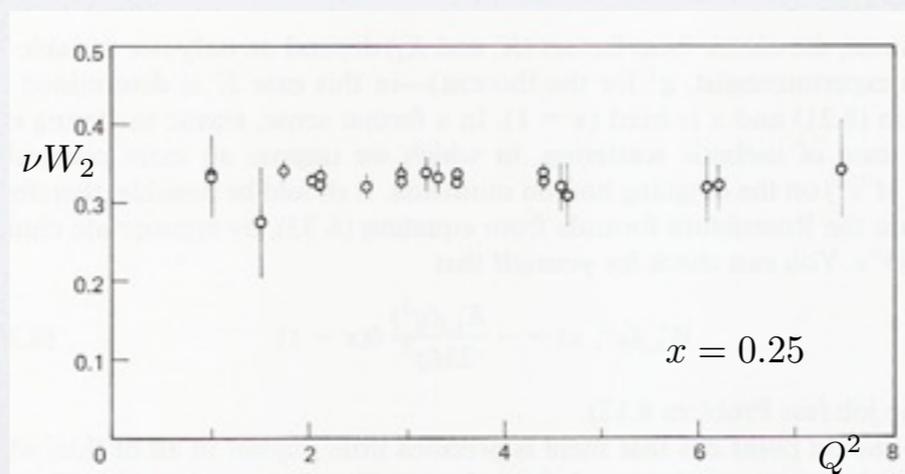
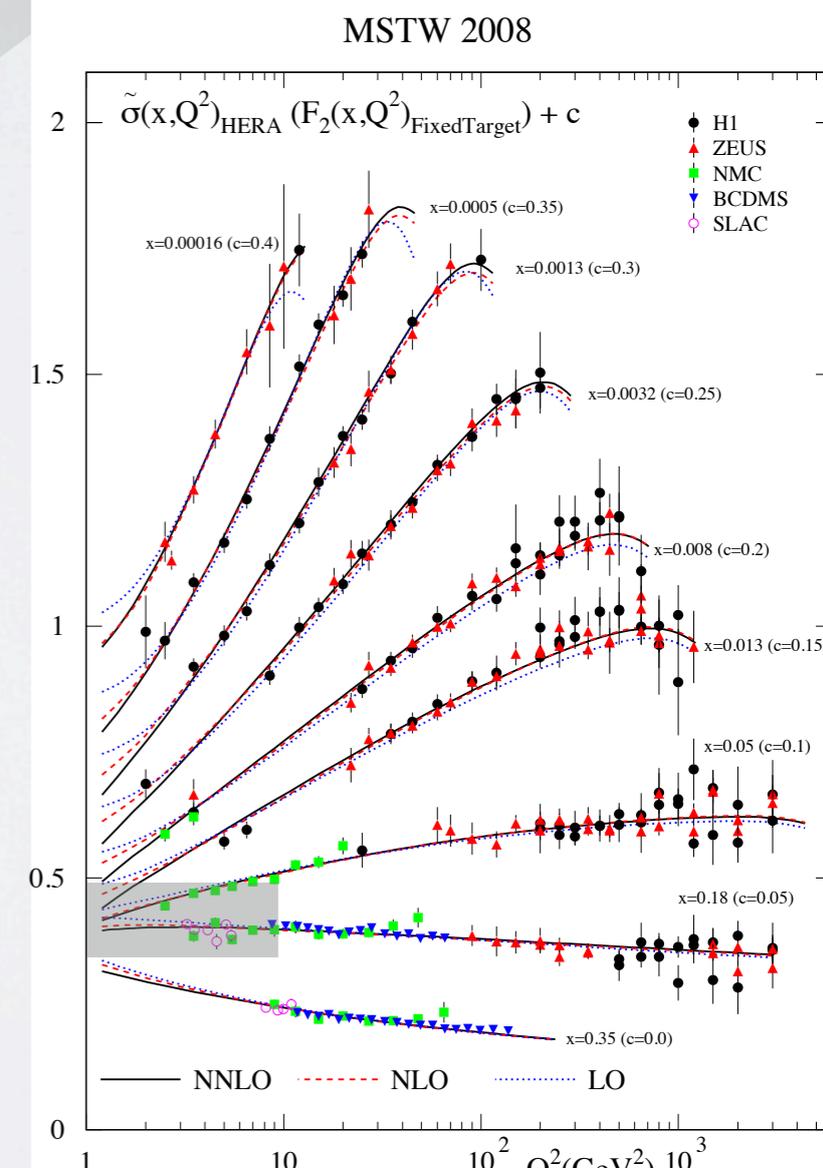


fig. 9.6 Scaling behavior of the structure function  $W_2$  in deep inelastic scattering



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